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## Stochastic Programming Approach versus Estimator-Based Approach for Sensor Network Design for Maximizing Efficiency

### Reference

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### ABSTRACT

The measurement technology with sensors plays a key role in achieving efficient operation of the process plants, and optimal sensor placement is very important in this endeavor. The focus of the current work is on the development of sensor placement algorithms to obtain the numbers, locations, and types of sensors for a large-scale process with the estimator-based control system. Two sensor placement algorithms are developed and investigated. In one algorithm, dynamics in the process efficiency loss that are due to the estimator-based control system that receives measurements from a candidate sensor network are explicitly accounted for. For a large-scale process with a large number of candidate sensor locations, this approach leads to a computationally expensive mixed integer nonlinear programming problem. In another algorithm, the estimation error is accounted for in terms of probability distributions, and therefore, a stochastic programming approach is used to solve the sensor placement problem. A novel algorithm called BONUS is used to solve the problem. The developed sensor placement algorithms are implemented in an acid gas removal unit as part of an integrated gasification combined cycle power plant with precombustion carbon dioxide capture. In this article, we compare and contrast these two sensor placement algorithms by evaluating the efficiency loss of the optimal sensor network synthesized by each of these algorithms along with their computational performance.

### Keywords

sensor placement, advanced power systems, BONUS algorithm, stochastic optimization

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## Nomenclature

$A$	= process transition matrix
$B$	= input matrix
$b$	= scalar budget, \$
$C$	= measurement matrix
$c_i$	= cost of individual sensor, \$
$K$	= steady state Kalman gain matrix
$l$	= number of total measurements
$N_{CS}$	= set of candidate measurements
$P$	= error covariance matrix
$Q$	= process noise covariance matrix
$R$	= measurement noise covariance matrix
$U$	= input
$u_{cont}$	= control input
$u_d$	= disturbance
$v$	= measurement noise vector
$w$	= process noise vector
$\hat{x}$	= vector of estimated states
$x_{(act)}$	= vector of actual states
$y$	= vector of noisy measurements
$\hat{y}$	= vector of estimated variables
$\beta$	= decision variable vector of binary numbers (0 and 1)
$\epsilon(t)$	= deviation of the controlled variable from set point
$\eta$	= efficiency

## Introduction

Advanced sensors play an important role in smart manufacturing as it employs computer control and high levels of adaptability. Sensors play a key role in the monitoring and control endeavor through the quantification of the dynamic state and structural integrity (surface strain, corrosion, refractory degradation) in the various components. The sensors in a chemical or power plant operate under harsh temperature and pressure and flow environments including corrosive and erosive conditions, which challenge sensor reliability and robust plant operation to meet performance and environmental targets. Sensor selection and layout of a plant are, therefore, important for real-time multidimensional mapping of key parameters to ensure an efficient, reliable, and safe operation. Sensor deployment design is governed by the following broad considerations: (1) the number, placement, measurement type, actuation, and networking of information of sensors must be optimally designed so as to reduce sensor and plant cost while maximizing sensor information and the effectiveness of sensors; (2) not all areas of the plant are amenable to hardware sensor access that is either due to physical limitations or a harsh environment; therefore, innovative sensing strategies are needed to supplement hardware sensing; (3) the sensors are subject to uncertainty in the measured data, which needs to be accounted for; (4) sensor network should be such that it can be easily modified by including sensors as new types of sensors become available or removing sensors as needed. Sensors can not only increase observability and hence control but can also be used to improve the efficiency of the plant. This is the focus of the current endeavor.

During the last 30 years, there have been a large number of articles published defining several measures of sensor location for state estimation in process plants using stationary sensor placement. When the sensor location is fixed, the optimality criterion for the parameter identification is based on scalar measures of the Fisher information [1–3] using local sensitivity information. For the combined problem of determining sensor location and parameter estimation, optimality criteria based on the error covariance matrix [4–9] (and the observability matrix, or the observability Gramian) [10,11] are commonly used. In addition, there are contributions that take into account measurement cost and sensor failure in addition to process information. Ali and Narasimhan [12] introduced a new concept considering reliability for sensor placement by using the probability of sensor failure in addition to observability and measurement redundancy. Bagajewicz [13] and Chmielewski, Palmer, and Manousiouthakis [14] present techniques subject to data-reconciliation-related constraints and related to the minimization of cost. Muske and Georgakis [15] present a sensor location technique that trades off between measurement cost and process information. However, these techniques are restricted to linear systems. The approaches in the literature related to nonlinear systems [11,16–18] are derived from the linearization of the systems and are limited to low-order systems.

The work specific to optimal sensor placement in highly nonlinear systems, especially for power systems, includes the following. Seenumani et al. [19] developed a nonlinear programming problem for sensor deployment in an integrated gasification combined cycle (IGCC) system gasifier model, which they solved using an outer-approximation-based algorithm with the intention of minimizing sensor cost subject to the estimation accuracy of a sensor set. Sensor placement for maximizing the degree of observability of a nonlinear system has been investigated by Singh and Hahn [11]. Nguyen and Bagajewicz [20,21] have developed an equation-based tree search method for designing nonlinear sensor networks. Wouwer et al. [1] have investigated sensor placement in a nonlinear distributed parameter model of a catalytic fixed-bed reactor. Sensor placement in a nonlinear convection-diffusion-reaction process has been studied by Alonso et al. [2]. Nonlinear observability functions have been considered as a criterion for sensor placement by Georges [22]. The use of a geometric approach for determining the degree of estimatability for nonlinear systems has been investigated by Lopez and Alvarez [16].

There are relatively fewer works in the literature that have investigated the economic impact of the sensor network—not necessarily from the cost of the sensor network perspective but rather from the perspective of the impact of the sensor network on the plant economics. In a series of works, Bagajewicz and his co-workers have investigated the sensor network from the perspective of the economic value of precision [23] (and data reconciliation and instrumentation upgrade [24,25]). Impact of the sensor network on the operational profit has been considered by Nabil and Narasimhan [26] for developing their sensor placement algorithm.

There are a handful of works in the existing literature on sensor placement for maximizing process efficiency. While the definition of process efficiency can vary depending on the plant, in this article, the term ‘process efficiency’ is used to mean a measure that affects a plant’s profitability. Because the sensor network is used to estimate the controlled variable, estimation error in the controlled variables can lead to a loss in process efficiency because the controlled variables are not maintained anymore at their optimal set points. The two approaches that currently exist in the literature for sensor network design for maximizing the efficiency of nonlinear systems have been proposed by the authors of this article. In a series of articles, Bhattacharyya and his co-workers have developed steady-

state and dynamic approaches where the impact of the sensor network on the efficiency in an estimator-based control system is directly taken into account [27–29]. These algorithms are applied to the acid gas removal (AGR) unit of the IGCC plant, which is a highly nonlinear system. The other approach has been developed by Sen, Sen, and Diwekar [30] where trade-offs between information, efficiency, and cost are taken into account for sensor placement under uncertainty in a nonlinear system. The approach is based on stochastic optimization. The last two are the only approaches proposed for sensor placement for maximizing efficiency. The first one considers the estimator dynamics and resulting errors, and the second approach includes stochasticity of estimation errors. The focus of this article is to compare these two approaches for efficiency maximization for a real-world case study from a subsection of the IGCC system. We compare the efficiencies obtained, sensors selected, and computational efficiencies of the two approaches.

This article is organized as follows. The section titled “Estimator-Based Approach for Sensor Placement” describes the estimator-based approach; this is followed by a section on the stochastic programming approach. The AGR case study is presented in the section titled “The AGR Case Study.” The final two sections contain results and discussions and a summary of the article.

## Estimator-Based Approach for Sensor Placement

In the estimator-based approach considered here, an optimal Kalman filter (KF) is used for estimating the states by using the measurements from the sensor network. Control actions, i.e., values of the manipulated inputs, depend on the estimated values of the controlled variables. Thus, estimation error in controlled variables can lead to suboptimal values of the manipulated variables. The values of the manipulated inputs, in turn, affect the plant efficiency. Therefore, the sensor network affects the plant efficiency. It should be noted that the estimation error in all controlled variables does not necessarily lead to loss in plant efficiency from the economic perspective. If a given set of controlled variables has been optimally selected by considering its impact on the plant economics, and the operating conditions are optimal, then the estimation error in that set of controlled variables would lead to a loss in efficiency. Thus, the overall objective of the approach is to minimize the deviation from the optimal efficiency,  $\eta_{opt}$ , that could have been achieved if the plant was optimally operated with no estimator or measurement errors. In the estimator-based control system shown in Fig. 1, the noisy measurements,  $y_\beta$ , from the sensor network are utilized by the filter to estimate the controlled variables and monitoring variables. Estimated values of the controlled variables are then used by the controllers to calculate the control action. Thus, estimation error in these controlled variables would lead to a loss in the efficiency of the process. Equations corresponding to Fig. 1 for a specific set of sensors are shown in Table 1.

The sensor network design (SND) objective is to minimize the deviation between  $\eta_{opt}$ , the maximum efficiency, and  $\eta(x_{act}, \beta)$ , the actual efficiency, i.e., the actual efficiency of the plant with a given sensor network and with due consideration of measurement and estimation errors. The SND optimization problem is given by the following:

$$\begin{aligned} & \text{Min}(\eta(x_{act}, \beta) - \eta_{opt})^2 \\ & \text{s.t.} \\ & \quad Ax_{act} + Bu + w = 0 \end{aligned}$$

FIG. 1

Schematic of the estimator-based control system.

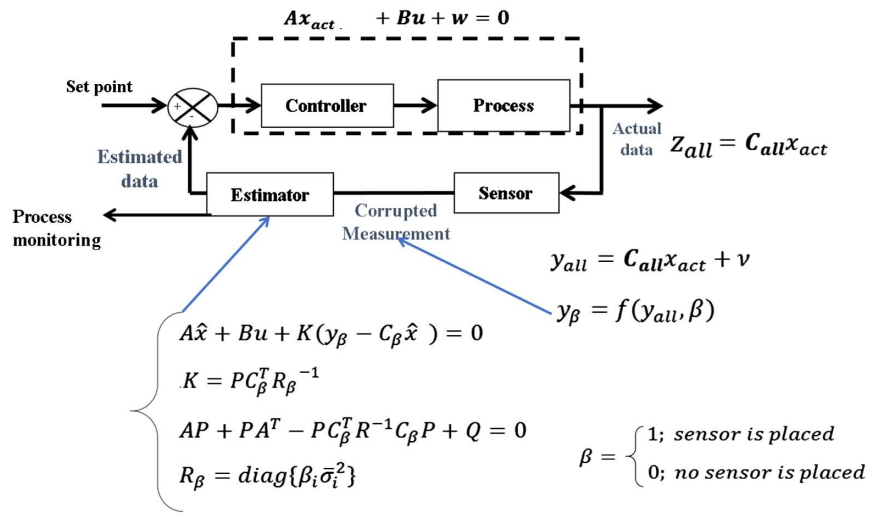


TABLE 1

Equations for estimator-based approach applied to the blocks in Fig. 1.

Process Model	Measurement Model
$\frac{dx_{act}}{dt} = Ax_{act} + Bu + w$ (1)	$y = Cx_{act} + v$ (2)
Estimator (KF)	Comparator
$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + K(y - C\hat{x})$ (3)	Estimated measurements: $\hat{y}_{cont,est} = C_{cont}\hat{x}$ (8)
$\frac{dP}{dt} = -PC^T R^{-1} CP + PA^T + AP + Q$ (4)	Error function: $\varepsilon(t) = y_{cont,set} - \hat{y}_{cont,est}$ (9)
$K = PC^T R^{-1}$ (5)	Controller (proportional only)
$Q = E[ww^T]$ (6)	Control action: $\frac{du_{cont}}{dt} = K_c \frac{d\varepsilon(t)}{dt}$ (10)
$R = E[vv^T]$ (7)	

$$y_\beta = C_\beta x_{act} + v_\beta \quad (11)$$

$$AP + PA^T - PC_\beta^T R_\beta^{-1} C_\beta P + Q = 0$$

$$K = PC_\beta^T R_\beta^{-1}$$

$$A\hat{x} + Bu + K(y_\beta - C_\beta\hat{x}) = 0$$

$$C_\beta = [C_{ij}]_{\beta_i \neq 0}; \quad v_\beta = [v_i]_{\beta_i \neq 0}$$

$$\sum \forall i c_i \beta_i \leq b$$

$$\beta_i = 0, 1 \quad \forall i \in N_{cs}$$

$$|y_{ma,act} - C_{ma}\hat{x}| < tol_1$$

$$j = 1, \dots, n; \quad j = 1, \dots, l;$$

In the previous formulation,  $\beta$  is the set of integer variables, while the remaining variables are continuous. Therefore, the SND problem is a mixed integer nonlinear programming (MINLP) problem. For all feasible combinations of sensors, it is difficult to satisfy the equality constraints. The optimization problem could be successfully solved by a sequential solution approach where the IP problem corresponding to the sensor selection is solved by a genetic algorithm (GA). The underlying nonlinear programming problem is solved by a sequential modular approach where an appropriate tear stream is selected so that the underlying nonlinear equality constraints can be satisfied reliably as the integer variables are changed. Additional details about the solution approaches can be found in Paul et al. [27].

## Stochastic Programming Approach for Sensor Placement

In the stochastic optimization/stochastic programming approach to the sensor placement problem, we consider uncertainties (errors) in measurement by sensors provided by manufacturers. The objective function then becomes the expected value of efficiency maximization, which is a nonlinear, stochastic, mixed integer problem, as shown in the following:

$$\max \beta_j [E(\eta(\beta_j, Y_j))]$$

$$\text{Subject to } \sum_{j=1}^{S_{out}} C_j \beta_j \leq b \quad j = 1, 2, \dots, S_{out} \tag{12}$$

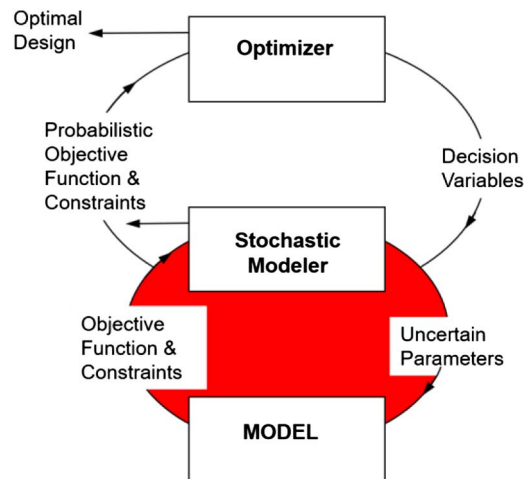
$$Y_j = P(\beta_j, x_i) \quad j = 1, 2, \dots, S_{out} \quad \beta_j \in (0, 1) \tag{13}$$

Here,  $E$  refers to the expected value of the objective function. The objective function is efficiency expressed as  $\eta$ , which is a function of sensor location  $j$ .  $\beta_j$  is 1 if the sensor is placed and is 0 if the sensor is not placed.  $C_j$  is the cost of the sensor  $j$ , and  $b$  is the budget.  $Y_j$  are the intermediate and output variables;  $x_i$  are the input variables.  $P$  is the process function (mass and energy balances) that relates input variables to intermediate and output variables.  $S_{out}$  denotes the maximum number of sensors. If sensors are placed at none of the locations and all of them are measured through virtual sensing, then the plant thermal efficiency is lowest because of high measurement uncertainties. As sensors are placed at these locations, the variability in the measurement of the intermediate variables reduces, leading to an increase in efficiency.

**Fig. 2** represents the generalized solution procedure for the stochastic programming problem where we need to find the probabilistic objective function and constraints. The stochastic modeler in **Fig. 2** uses probability distributions of the uncertain variables to generate samples that are passed to the deterministic model to find the probabilistic values of the objective function (expected value in our case) and constraints at each of the sample iterations. Thus, for each optimization iteration, we need to iteratively use the stochastic model with the sampling loop to find the values of the objective function and constraints. The

**FIG. 2**

Generalized approach to stochastic optimization problems.

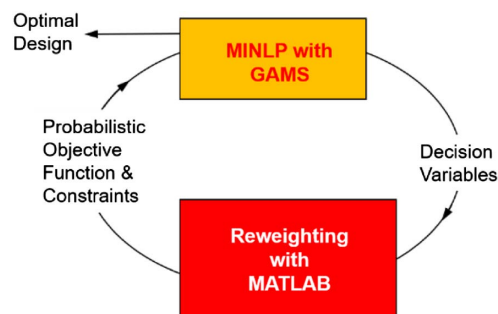


optimizer then uses these values and the derivative information to determine if the decision variables are at an optimum or not. If not, it continues the iterations. In this work, we will use efficient algorithms based on the BONUS algorithm proposed by Sahin and Diwekar [31,32].

BONUS is a novel algorithm for stochastic nonlinear programming (SNLP) problems [31,32]. Traditional SNLP methods rely on improving the probabilistic objective function by repeating the evaluation for each sample for every iteration, but in BONUS, instead of running the model for given samples in every iteration, the information of the output distribution for the base sample set is used to estimate the subsequent output distributions. An initial, uniform base distribution is generated, and the model is run to determine the output distribution. For better uniformity of samples, efficient sampling techniques like Hammersley Sequence Sampling or Latin Hypercube Hammersley Sampling can be used [33,34]. In the subsequent iterations, the model is not re-run. Instead, the reweighting approach is applied to approximate the probabilistic behavior of the new output distribution. This is implemented in Fig. 3 where there is no stochastic modeling loop in the figure. This concept of reweighting is central to BONUS and is incorporated in the proposed algorithm. It has been recently shown that BONUS reduces computational time by 99.7 % (as compared to traditional stochastic programming approach shown in Fig. 2) when used in the sensor placement problem for advanced power systems [18,30].

**FIG. 3**

BONUS approach to stochastic optimization.



In BONUS, initially, the stochastic model (internal loop in Fig. 2) is run with base distribution. The base distribution in the case of sensor placement is related to the measurement uncertainties when the sensor is not placed, i.e., using virtual sensing. Virtual sensing is the process by which an estimated value of a variable is calculated through mathematical modeling without placing any direct physical means of measurement such as a sensor [35]. Virtual sensing is expected to have a large error (in our case study, we have used 25 % error as this error should be much larger than sensor errors). The comprehensive model of the process or power plant is used to simulate steady-state performance. This nonlinear model is used to estimate the set of unmeasured variables using the data acquired from the process variables directly measured through the network of sensors physically deployed within the plant. A set of  $N_s$  input variable operating conditions are generated using Hammersley sequencing, a low-discrepancy sampling method, to generate the uniform sample space. Then, these samples are propagated through the model to generate a corresponding vector of points for  $S_{out}$  intermediate and output process variables  $Y_j$ . The resulting vector set of intermediate and output variables can be used to capture the nonlinear effects of the plant, as well as the variability of downstream variables resulting from a uniformly distributed set of input variable sample points.

Let  $f_0(x_i)$  be the probability density function (PDF) associated with the base input distribution for input variables  $x_i, i = 1, 2, \dots, S_{in}$ , respectively. Following the simulation of the IGCC process at iteration  $t = 0$ , let  $f_{y_0}(y_i)$  and  $F_{y_0}(y_i)$  be the base PDF and cumulative distribution function associated with the intermediate and output variables  $y_j, j = 1, 2, \dots, S_{out}$ , where  $y_j = P(x_i, i = 1, 2, \dots, S_{in}), j = 1, 2, \dots, S_{out}$  is the nonlinear transformation from each input variable,  $x_i$ , to the downstream variable,  $y_j$ .

Next, consider when a new input distribution is defined, such as when a sensor is placed at the location of an input variable. This new input distribution is a normal distribution defined using the accuracy of sensors given by manufactures ( $3\sigma = \pm accuracy$ ). The redefined distribution  $f_t(x_i)$  at iteration  $t$  is used to create a set of weights.

$$W_t(x_i) = \frac{f_t(x_i)}{f_0(x_i)}, \quad i = 1, 2, \dots, S_{in} \quad (14)$$

This gives the likelihood ratio between the redefined and base distributions. Given that the input variables act independently, these weights are used to construct the resulting distribution for the downstream intermediate or output variables at iteration  $t$  by multiplying the associated weights  $W_t(x_i)$  with the base distribution  $f_{y_0}(y_j)$ ,

$$f_{y_t}(y_j) = f_{y_0}(y_j) \prod_{i=1}^{S_{in}} (1 + \gamma_{ij}(W_t(x_i) + 1)), \quad j = 1, 2, \dots, S_{out} \quad (15)$$

where  $\gamma_{ij} = 1(0)$  if variable  $y_j$  is (is not) downstream of  $x_i$ . The distribution  $f_{y_t}(y_j)$  is normalized using the following equation:

$$\hat{f}_{y_t}(y_j) = \frac{f_{y_t}(y_j)}{\sum_{l=1}^{N_s} f_{f_t(n)}((y_j(n+1) - y_j(n-1))/2)} \quad (16)$$

This reweighting approach can also be used when a sensor is placed at the location of an intermediate process variable to construct the resulting change in distributions of corresponding downstream variables.



Because efficiency is one of the output variables, say  $y_m$ , the expected value of efficiency can be calculated using Eq 17 as follows:

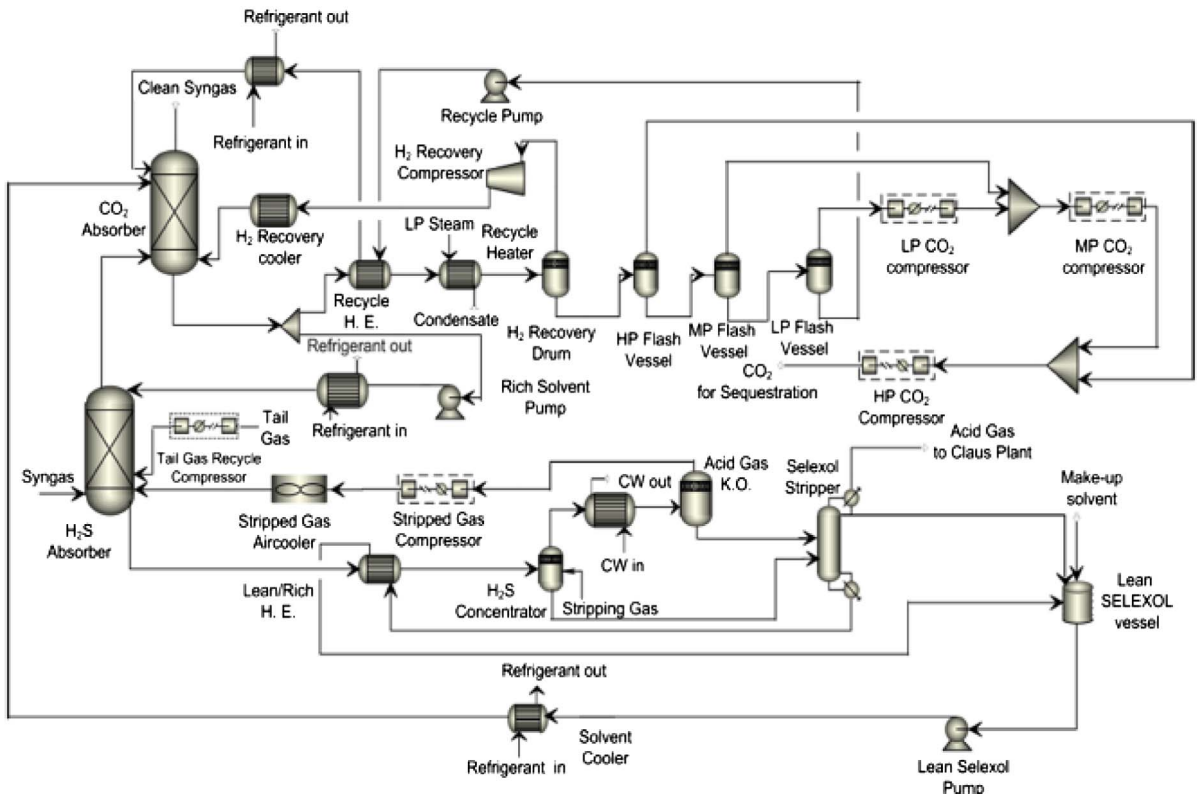
$$E(y_m) = \sum_{n=1}^{N_s} y_t(y_m(n))y_m(n) \quad (17)$$

In this work, the sensor placement problem is a mixed integer nonlinear stochastic problem. We are using the General Algebraic Modeling System (GAMS) to solve the mixed integer nonlinear programming (MINLP) problem, and the reweighting scheme is programmed in MATLAB (MathWorks, Natick, MA). Please refer to Fig. 3.

## The AGR Case Study

Fig. 4 shows the AGR unit where both algorithms are tested. The plant model is part of an integrated gasification combined cycle (IGCC) unit [36] where the Selexol solvent is used to capture hydrogen sulfide ( $H_2S$ ) selectively in the lower tower ( $H_2S$  absorber), whereas carbon dioxide ( $CO_2$ ) is captured in the upper tower ( $CO_2$  absorber). A semi-regenerated solvent is used to capture the bulk of the  $CO_2$  in the  $CO_2$  absorber where also a thermally regenerated solvent stream from the stripper is used for final polishing of the acid gas stream before it is sent to the gas turbine. The partially loaded solvent from the absorber

FIG. 4 Schematic of the acid gas removal unit.



bottom is split into two streams. Most of the dissolved hydrogen ( $H_2$ ) in a portion of the loaded solvent stream is removed in the  $H_2$  recovery drum and recycled back to the  $CO_2$  absorber. This stream then goes through a series of separators where  $CO_2$  is removed at different pressure levels by pressure swing. The solvent is finally chilled by a refrigerant before sending it back to the absorber. The other portion of the  $CO_2$  absorber bottom stream goes to the  $H_2S$  absorber where the tail gas from the Claus unit is also sent to the bottom of the tower along with the syngas. The fully loaded solvent from the bottom of the  $H_2S$  absorber first goes through a lean/rich heat exchanger where heat is recovered from the stripper bottom stream to heat up the solvent. The heated solvent is then sent to the  $H_2S$  concentrator that strips off a significant portion of the dissolved  $CO_2$  in the solvent. The  $CO_2$  is then compressed and recycled back to the  $H_2S$  absorber. The solvent from the  $H_2S$  concentrator then goes to the stripper where it is thermally regenerated. The stripper off-gas is rich in  $H_2S$  and is sent to the Claus unit. The thermally regenerated solvent first undergoes heat recovery and is then chilled before sending it back to the  $CO_2$  absorber. This process has significant mass and energy interaction and is energy intensive, so it is desired that the efficiency penalty that is due to the measurement system is minimized. Therefore, the system serves as a very good test case for the sensor placement problems. The process has 1,505 state variables. More details about this unit can be found in the work of Bhattacharyya, Turton, and Zitney [36].

The following types of candidate sensors were considered in this work: temperature, pressure, composition, and flow sensors. Different types of flow sensors are considered for the liquid and gas phases. Flow sensors are further classified based on the range of the flowrate. Table 2 shows the types of sensors, % inaccuracy, range, and the cost used in the work [27]. It should be noted that the cost of the sensors includes the installation cost along with the cost of the measuring device, transmitter, and other accessories.

## Results and Discussions

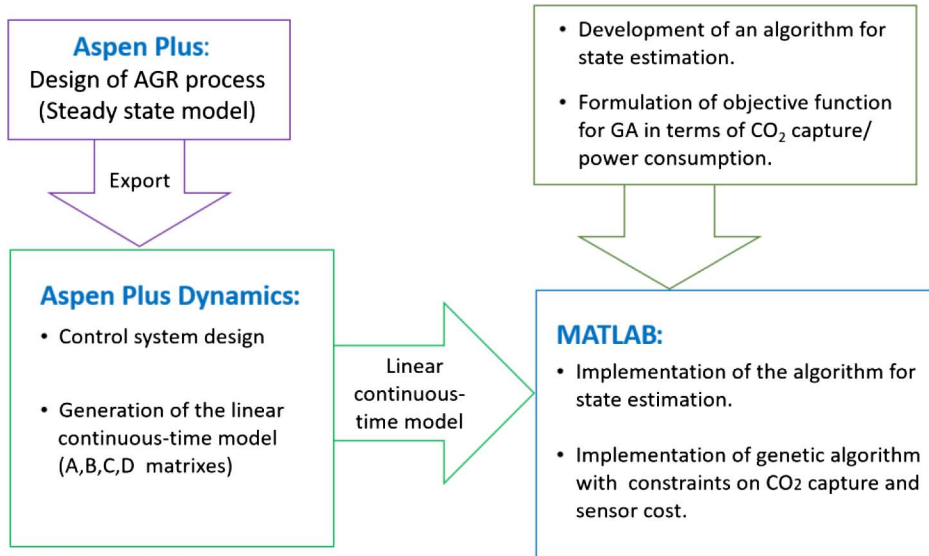
Fig. 5 presents the estimator-based approach to solve the sensor placement problem for efficiency maximization of the AGR process. The model for AGR is developed in Aspen Plus (Aspen Technology, Bedford, MA) and exported to Aspen Plus Dynamics (Aspen Technology, Bedford, MA) where the control system is designed, and a continuous-time linear model is generated by linearizing the nonlinear model around the nominal operating condition. The estimator-based approach for sensor placement is implemented in MATLAB

**TABLE 2**

Cost and accuracy of sensors used in the AGR example [27].

Types		Inaccuracy	Range	Cost
Flow sensor (includes plate, flanges, flanged meter, and transmitter)	Gas phase	$\pm 0.25\%$ to $\pm 0.5\%$ of actual flow	5–20 cm pipe	\$3,400
	Gas phase		20–50 cm pipe	\$7,000
	Liquid phase		1–35 cm pipe	\$5,300
	Liquid phase		70–100 cm pipe	\$14,000
Pressure measurement device (includes the transmitter)		0.1 %–1 % of span	0 to 6.9 MPa	\$2,500
Temperature (thermocouple integrated with a transmitter)		$\pm 1^\circ C$ to $\pm 2.8^\circ C$	$-174^\circ C$ to $500^\circ C$	\$1,000
$H_2S$ analyzer 0–500 ppm (includes installation cost)		1 % of full scale	0 to 500 ppm	\$70,000
$CO_2$ analyzer (NDIR analyzer, includes installation cost)		1–2 % of full scale	\$10,000	\$10,000

**FIG. 5** Estimator-based approach for sensor placement for AGR process.



where the linear model generated from Aspen Plus Dynamics is used. The efficiency of the process is considered to be the amount of CO<sub>2</sub> captured per unit of power consumption.

For the stochastic optimization/stochastic programming approach. The BONUS-based MINLP is used. In the first part, Hammersley sequence sampling is used to generate 800 samples of input parameters affecting the flowsheet. This required five hours of central processing unit (CPU) time. These samples are propagated through the AGR flowsheet, and the output parameters of interest are collected. This forms the base distribution for reweighting. The approach shown in Fig. 3 is used to obtain optimal sensor placement where the top MINLP problem was solved using GAMS, but the reweighting is done in MATLAB.

The following approach was adopted for identifying the candidate sensors in the AGR process. For all the towers (i.e., H<sub>2</sub>S and CO<sub>2</sub> absorbers, stripper, and H<sub>2</sub>S concentrator), only one  $T$  and  $P$  for each tray are considered as candidate sensor locations because the liquid and vapor phases in each tray have the same temperature as these trays are modeled as equilibrium stages. For the heat exchangers, mass/molar flowrate and composition sensors are considered only at the outlet as they remain unchanged through the heat exchanger. However, temperature and pressure both at the inlet and outlet are considered as candidates because they change across the heat exchanger. For compressors, both inlet and outlet pressure and temperatures across the compressor are considered as candidates. Both inlet and outlet pressures of the pumps and valves are considered as candidates. Additional details about candidate sensor locations for this unit can be found in the work of Paul et al. [27,28]. In total, there are 163 candidate measurements that were identified. Out of these, 46 candidate sensors are identified in the equipment items, while remaining candidate sensors are in the process streams in the AGR unit. There are 14, 12, 7, 2, and 11 candidate sensors in the H<sub>2</sub>S absorber, CO<sub>2</sub> absorber, H<sub>2</sub>S concentrator, acid gas knockout drum, and solvent strippers, respectively. In the process streams, there are 47, 33, 18, 3, and 16 candidate sensors for temperature, pressure, flow, H<sub>2</sub>S concentration, and CO<sub>2</sub> concentration, respectively.

**TABLE 3**

Comparison of estimator-based versus BONUS-based approach for sensor placement.

Case Study Number	Cost of Sensor Network	Efficiency by BONUS-Based Algorithm, kmol CO <sub>2</sub> /MWh	Efficiency by Estimator-Based Approach, kmol CO <sub>2</sub> /MWh	CPU Time BONUS*	CPU Time EBA	Number of Optimal Sensors BONUS	Number of Optimal Sensors EBA
1	\$431,900	766.2	766.01	1.8 s	4 hr 11 min	75	11
2	\$322,600	766.2	766.01	1.5 s	4 hr 47 min	66	11
3	\$229,400	766.2	766.01	2.0 s	5 hr 2 min	64	11
4	\$187,900	766.2	766.01	2.0 s	7 hr 22 min	62	11
5	\$149,000	766.2	765.11	2.0 s	13 hr	56	11
6	\$118,700	766.2	762.64	1.7 s	18 hr	46	11
7	\$71,200	741.1	758.25	1.5 s	3 hr 16min	25	10
8	\$63,700	741.1	756.76	1.2 s	3 hr 18 min	25	10
9	\$60,200	741.1	752.04	1.7 s	3 hr 45 min	23	10
10	\$59,700	741.1	750.07	1.8 s	4 hr 47 min	24	–
11	\$42,500	741.1	742.86	1.5 s	5 hr 17 min	17	–

**Table 3** presents the comparison of the results of the estimator-based approach versus the BONUS-based approach for different sensor network costs. For the estimator-based approach, the case for the highest budget is solved first. For this case, the initial guess for the sensor network was random. More than 200 random initial guesses were evaluated. **Table 3** shows the best value that was obtained. Interestingly, most of the initial guesses converged to this solution. Because the efficiency  $\eta(x_{act}, \beta)$  reported in **Table 3** for the highest budget case is very close to the optimal efficiency,  $\eta_{opt}$ , the solution was found to be acceptable. For the lower budget cases, the initial population for GA is the same as the optimal sensor set obtained for the budget that is immediately higher.

**Table 3** shows that the results of the two approaches are comparable. However, the BONUS-based approach cannot differentiate small changes in the objective function as can be seen in **Table 3**. This is because the reweighting scheme provides an approximate value of the probabilistic objective function and constraint. The approximation is a function of the number of samples. The accuracy increases with the number of samples.

**Table 3** shows that as the budget is increased, it helps to increase the efficiency because of the higher estimation accuracy of the controlled variables as a result of the higher number of sensors or costlier sensors or both, especially the composition sensors. The estimation accuracy does not change much beyond a certain budget; therefore, the optimal efficiency,  $\eta_{opt}$ , and the actual efficiency,  $\eta(x_{act}, \beta)$ , become very close to each other for the estimator-based approach beyond a certain budget. It can be noted that there is a steeper decrease in the efficiency when the budget becomes lower than a certain value. It has been observed that as the budget is decreased below a budget, a composition sensor cannot be afforded as these sensors, especially the H<sub>2</sub>S concentration sensors, are considerably costlier than the other types.

**Table 4** shows the sensors selected by the two approaches for the sensor network cost of \$229,400. Although the optimal value of the efficiency is the same for both the approaches, the actual sensors selected are different (**Table 4**). This is because the problem has multiple solutions. Even though there are considerable differences between the sensor networks, one interesting thing that can be observed is that the BONUS-based approach selects one of the H<sub>2</sub>S concentration sensors (Sensor #126) that costs \$70,000, while the estimator-based approach selects seven CO<sub>2</sub> concentration sensors (Sensor #150, 151, 154, 155, 157, 160, 163) that also cost \$70,000 in total.

**TABLE 4**  
Optimal sensor locations.

Sensor Network Budget: \$230,000		
BONUS Based Algorithm	Estimator-Based Algorithm	Location Description
–	1	<sup>2</sup> H <sub>2</sub> S absorber temperature <sup>a</sup>
4	–	<sup>20</sup> H <sub>2</sub> S absorber temperature <sup>a</sup>
5	–	<sup>25</sup> H <sub>2</sub> S absorber temperature <sup>a</sup>
–	6	<sup>3</sup> H <sub>2</sub> S concentrator temperature <sup>a</sup>
7	–	<sup>5</sup> H <sub>2</sub> S concentrator temperature <sup>a</sup>
–	10	<sup>8</sup> CO <sub>2</sub> absorber temperature <sup>a</sup>
12	–	<sup>1</sup> Solvent stripper temperature <sup>a</sup>
13	13	<sup>3</sup> Solvent stripper temperature <sup>a</sup>
15	–	<sup>10</sup> Solvent stripper temperature <sup>a</sup>
–	19	CO <sub>2</sub> absorber feed inlet temperature
–	20	CO <sub>2</sub> absorber clean syngas outlet temperature
–	22	CO <sub>2</sub> absorber bottom outlet temperature
24	24	H <sub>2</sub> S absorber bottom outlet temperature
26	–	H <sub>2</sub> S concentrator bottom temperature
–	27	Solvent stripper bottom temperature
28	24	Lean solvent outlet temperature at the outlet of lean/rich exchanger
–	27	
–	30	H <sub>2</sub> recovery flash vessel temperature
–	31	HP flash vessel temperature
–	32	MP flash vessel temperature
–	37	Stripped gas cooler outlet temperature
38	–	Stripper feed temperature
–	39	Temperature of the steam to the stripper
40	–	H <sub>2</sub> recovery flash vessel temperature
–	41	H <sub>2</sub> recovery compressor discharge temperature
–	42	H <sub>2</sub> recovery cooler outlet temperature
–	43	MP CO <sub>2</sub> compressor inlet temperature
–	45	2nd LP CO <sub>2</sub> compressor inlet temperature
–	46	2nd LP CO <sub>2</sub> compressor outlet temperature
47	–	3rd LP CO <sub>2</sub> compressor outlet temperature
–	48	1st MP CO <sub>2</sub> compressor inlet temperature
–	50	2nd MP CO <sub>2</sub> compressor outlet temperature
–	52	1st HP CO <sub>2</sub> compressor inlet temperature
–	53	2nd HP CO <sub>2</sub> compressor inlet temperature
–	54	2nd HP CO <sub>2</sub> compressor outlet temperature
–	60	4th HP CO <sub>2</sub> compressor outlet temperature
–	62	3rd HP CO <sub>2</sub> compressor inlet temperature
–	65	<sup>16</sup> H <sub>2</sub> S absorber pressure <sup>a</sup>
–	68	Acid gas knockout drum pressure
–	69	<sup>3</sup> CO <sub>2</sub> absorber pressure <sup>a</sup>
–	71	<sup>15</sup> CO <sub>2</sub> absorber pressure <sup>a</sup>
–	73	<sup>3</sup> Solvent stripper pressure <sup>a</sup>
–	78	Rich solvent pump discharge pressure
–	83	Lean solvent pump outlet pressure
–	84	H <sub>2</sub> recovery drum inlet pressure
–	85	HP flash vessel pressure

TABLE 4 Continued

Sensor Network Budget: \$230,000		
BONUS Based Algorithm	Estimator-Based Algorithm	Location Description
–	86	MP flash vessel pressure
–	90	Stripped gas compressor discharge pressure
–	92	Solvent stripper feed stage pressure
–	93	H <sub>2</sub> recovery drum pressure
–	94	H <sub>2</sub> recovery compressor discharge pressure
–	96	HP compressor inlet pressure
–	100	LP compressor inlet pressure
–	102	2nd MP CO <sub>2</sub> compressor discharge pressure
–	103	3rd HP CO <sub>2</sub> compressor inlet pressure
–	106	Final HP CO <sub>2</sub> compressor inlet pressure
–	107	Tail gas recycle compressor discharge pressure
109	–	CO <sub>2</sub> absorber feed flow
–	112	H <sub>2</sub> S absorber bottom flowrate
–	115	Semi-lean solvent flowrate to recycle heat exchanger
–	118	Semi-lean solvent flowrate to the CO <sub>2</sub> absorber
–	119	H <sub>2</sub> S concentrator vapor outlet flowrate
–	120	Solvent stripper vapor outlet flowrate
–	121	Stripping steam to stripper flowrate
123	123	MP flash vessel outlet flowrate
–	125	
126	–	<sup>5</sup> H <sub>2</sub> S concentration in H <sub>2</sub> S absorber <sup>a</sup>
145	145	<sup>8</sup> CO <sub>2</sub> absorber CO <sub>2</sub> mole fraction in liquid phase <sup>a</sup>
–	150	CO <sub>2</sub> absorber bottom CO <sub>2</sub> mole fraction
–	151	H <sub>2</sub> S absorber bottom CO <sub>2</sub> mole fraction
–	154	HP flash vessel liquid outlet CO <sub>2</sub> mole fraction
–	155	MP flash vessel liquid outlet CO <sub>2</sub> mole fraction
–	157	H <sub>2</sub> S concentrator vapor outlet CO <sub>2</sub> mole fraction
–	160	Solvent stripper top vapor outlet CO <sub>2</sub> mole fraction
–	163	3rd HP CO <sub>2</sub> compressor CO <sub>2</sub> mole fraction

Note: <sup>a</sup> Superscript at the beginning of each location denotes stage number.

It has been found that on average the CPU time required for the estimator-based approach is 6 hours 37 minutes. On the other hand, the CPU time required to solve the sensor placement problem using the BONUS-based approach was an average CPU time of 1.722 seconds, getting significantly faster results than the estimator-based approach (even if we had to generate the initial samples that needed a CPU time of five hours. This needs to be done only once and the problem with any budget can be solved). For BONUS, the CPU time seems to be independent of the scale of the problem.

## Summary

Optimal sensor placement is an important problem in smart manufacturing as the cost of sensors and their installation can be prohibitive. Sensors can play an important role in improving the efficiency of the plant by reducing the error in measurements so that tight

control can be achieved. The problem of sensor placement for maximizing efficiency is relatively new in the literature. There are two approaches to this problem, namely, the estimator-based approach and the stochastic programming approach. This article compared the two approaches for solving the problem of sensor placement for maximizing efficiency in an IGCC power plant. We considered the AGR flowsheet in the IGCC system for this analysis. Although, the two approaches have been shown to give comparable results as far as maximizing efficiency is concerned. The stochastic programming approach based on the BONUS algorithm is computationally more efficient. However, the BONUS-based approach cannot distinguish small changes in the objective function. The problem is also shown to have multiple solutions. In the future, we will be using this approach for real-time sensor placement for power systems. The other applications where this approach could be used are in water distribution networks, air pollution monitoring, and drone-based sensing.

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